ANSWER TO PROF. HALL'S QUERY (SEE P. 48) BY PROF. H. T. EDDY.

Take the well known Eulerian integral

$$\int_0^{\frac{\pi}{2}} \sin^p\varphi \cos^q\varphi \, d\varphi = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}; \; p=\tfrac{1}{2}, \; q=0.$$

$$\therefore \qquad u = \frac{\Gamma(\frac{3}{4})\,\Gamma(\frac{1}{2})}{2\Gamma(\frac{5}{4})} = \frac{\sqrt{\pi}}{2}\cdot\frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}.$$

Again, $\dfrac{[\Gamma(n)]^2}{\Gamma(n-m)\Gamma(n+m)} = \left(1-\dfrac{m^2}{n^2}\right)\left(1-\dfrac{m^2}{(n+1)^2}\right)\left(1-\dfrac{m^2}{(n+2)^2}\right)\ldots.$

In this equation make $m=\frac{1}{4}$, and, first, put $n=\frac{3}{4}$ and then $n=\frac{5}{4}$. Dividing the first result by the second we get

$$\frac{\Gamma(1)\,\Gamma(\frac{3}{2})\,[\Gamma(\frac{3}{4})]^2}{\Gamma(\frac{1}{2})\,\Gamma(1)\,[\Gamma(\frac{5}{4})]^2} = \frac{[\Gamma(\frac{3}{4})]^2}{2[\Gamma(\frac{5}{4})]^2}.$$

$$\therefore \qquad u = \left[\frac{\pi}{2}\cdot\frac{\frac{25}{24}\cdot\frac{81}{80}\cdot\frac{169}{168}\cdot\frac{289}{288}\cdot\frac{441}{440}\cdot\frac{625}{624}\cdot\frac{841}{840}\cdot\ldots\ldots}{\frac{8}{9}\cdot\frac{49}{48}\cdot\frac{121}{120}\cdot\frac{225}{224}\cdot\frac{361}{360}\cdot\frac{529}{528}\cdot\frac{729}{728}\cdot\ldots\ldots}\right]^{\frac{1}{2}},$$

which is in a form convenient for computation from a table of logarithms, and especially it is easy to obtain the logarithms of the fractions at the right of those given as they are simple tabular differences. Fifteen terms or perhaps less, are sufficient to obtain $u = 1.198\ldots.$

ANSWER TO MR. HEAL'S QUERY. (SEE PAGE 64.)

[T. P. STOWELL of Rochester, N. Y., writes: "Perhaps it would interest some of your readers to reprint a paper published in the Math. Repository (Vol. I, 2nd series) in 1806." As our space will net permit the publication of the paper referred to, in full, we give the following extract, and subjoin the Construction of a polygon of 17 sides, sent by Mr. Stowell, and credited to Leybourn's Math. Repository, 1818.]

"Mr. Gauss, of Brunswic, published at Leipsic, in 1801, a work called *Disquisitiones Arithmeticae.* In this work the author announces, that we may always inscribe a regular polygon of 2^n+1 sides in a circle, when n is a whole number, and 2^n+1 a prime number.

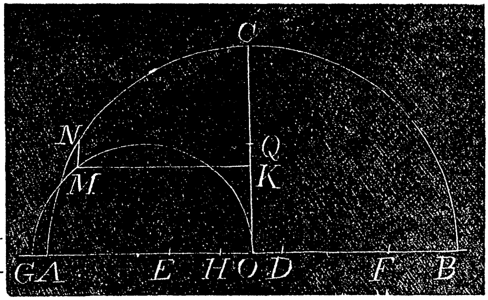
"Mr. Legendre has given to the French National Institute a demonstration of this very curious proposition in the case when the number of sides is 17. It is founded on these two lemmas:

Lemma I. Let a be the arch of a circle, m and n two whole numbers then $\qquad 2\cos ma\cos na = \cos(m-n)a + \cos(m+n)a.$

Lemma II. Let a be the nth part of a whole circumference n being a whole number, then

$$\cos a + \cos 2a + \cos 3a + \ldots. + \cos na = 0."$$

"TO CONSTRUCT a regular polygon of seventeen sides in a circle.



Draw the radius CO at right-angles to the diameter AB: On OC and OB, take OQ equal to the half, and OD equal to the eighth part of the radius: Make DE and DF each equal to DQ, and EG and FH respectively equal to EQ and FQ; take OK a mean proportional between OH and OQ, and through K, draw KM parallel to AB, meeting the semicircle described on OG in M; draw MN parallel to OC, cutting the given circle in N—the arc AN is the seventeenth part of the whole circumference."

PROBLEMS.

162. SELECTED.—Given

$$x^2 + xy + y^2 = 37, \quad (1)$$
$$x^2 + xz + z^2 = 49, \quad (2)$$
$$y^2 + yz + z^2 = 61, \quad (3)$$

to find x, y and z by quadratics.

163 BY PROF. ORSON PRATT, SEN.— Resolve the first member of the general Cubic Equation, $x^3 + px^2 + qx = -r$, into three factors, such that, when their signs are changed, their sum shall equal p; when their signs are unchanged the sum of their products, taken two and two, shall equal q; and when their signs are unchanged, their continued product shall $= -r$. Or, in other words, find the forms of the three roots in terms of x and the coefficients.

164. BY PROF. W. W. BEMAN, ANN ARBOR, MICH.—From any two points to draw two lines which shall meet in the circumference of a given circle, and make equal angles with the tangent at the point of intersection.

165. FROM BOOLE'S DIFF. EQ'NS (by request).—Of the system of dynamical equations,

$$\frac{d^2x}{dt^2} + \frac{mx}{r^3} = 0, \quad \frac{d^2y}{dt^2} + \frac{my}{r^3} = 0, \quad \frac{d^2z}{dt^2} + \frac{mz}{r^3} = 0,$$

where $r = \sqrt{(x^2+y^2+z^2)}$, seven first integrals are obtained of which it is subsequently found that five only are independent. How many final integrals can hence be deduced without proceeding to another integration.